

FIGURE 7.

THE GEOMETRICAL THEORY OF HALOS—VI¹

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[Weather Bureau, Washington, D. C., September 1941]

PART 3. THE OPTICAL METEORS PRODUCED BY ICE CRYSTALS IN THE ATMOSPHERE

Among the innumerable crystalline forms produced by the condensation of water vapor in the atmosphere at temperatures below freezing, as illustrated, e. g., in frost-work and by snowflakes, there are two or three quite simple ones from which all the others may be built up, viz, hexagonal columns with or without pyramidal caps (complete or truncated) and hexagonal plates; the columns are sometimes capped with plates, and the pyramids may occur unattached to columns.

These elementary forms (figure 19) are frequently observed in snow and frost at low temperatures, especially in polar regions; they often are present in the atmosphere at the surface of the earth when a halo display is witnessed, and there is every reason to believe that it is some one or more of them, or simple combinations thereof, which ordinarily produce halos, and not the complicated crystal groups and patterns shown in general by snowflakes—in fact, the majority of authenticated halos do not require anything more complicated than a simple hexagonal right prism (column or plate).

The present investigation will therefore be restricted to hexagonal right prisms (in the form of either columns or disks), hexagonal right pyramids (complete or truncated), and simple combinations of these two forms.

From the six lateral faces and two bases of a hexagonal right prism, taken two at a time, may be formed 28 possible combinations. Of these combinations, one consists merely of the two bases, which form a refracting angle of 0° and do not produce any resultant deviation; 15 are combinations between lateral faces, of which 3 are between opposite faces and again form 0° angles, and 6 are between adjacent faces and form angles of 120° through which no transmission is possible; 6 are between alternate faces, and all form truncated 60° refracting angles; 12 are combinations of a lateral face with a base, forming in all cases a refracting angle of 90°.

To determine all the halos which a hexagonal right prism with plane bases is capable of producing, it is necessary to calculate, for each of all orientations of the prism in space, the images obtained by refraction through the six 60° angles between alternate lateral faces and through the twelve 90° angles between lateral faces and bases, together with the images formed by reflection (external, and internal with or without accompanying refraction) from the six lateral faces and two bases. The refracting edges of the 60° angles are parallel to the principal axis of the crystal, and those of the 90° angles are perpendicular to the principal crystal axis.

In pyramidal crystals, the triangular faces may, according to the laws of crystallography, have any one of several different inclinations to the principal axis of the crystal, these different values being connected by simple numerical relations. Unfortunately, the possible inclinations can be

¹ The previous papers have appeared in the MONTHLY WEATHER REVIEW as follows: I, 64:321-325, 1936; II, 65:4-8; III, 65:55-57; IV, 65:190-192; V, 65:301-302, 1937. The figures in the present paper are numbered consecutively with those in the earlier papers.

determined only by actual measurement of the particular one of them that some actual crystal happens to have; an ice crystal is a difficult thing to measure—few such measurements have been made, and they necessarily are more or less inexact. Hence the values of the possible, and of the actually occurring, inclinations are somewhat uncertain. The most probable value—one determined in part from the evidence of the halos themselves—seems to be the one deduced by Humphreys,² viz, $24^{\circ}51'$, and it

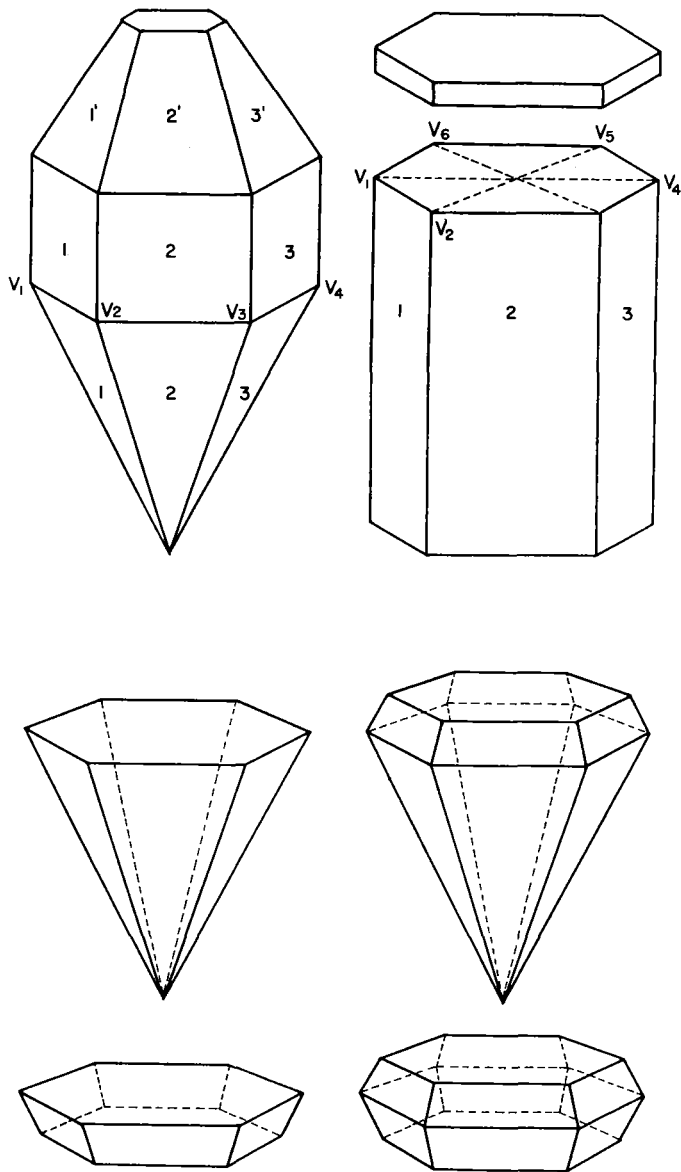


FIGURE 19.—Elementary Forms of Ice Crystals.

will be adopted here; the calculations for pyramidal crystals may easily be repeated by anyone who desires, with any other given value of this angle.

Then, in a crystal with a pyramidal element, the refracting angles through which transmission is possible (i. e., the dihedral angles less than $99^{\circ}31'$) are as follows (see figure 20):

² W. J. Humphreys, *Physics of the Air*, 3 ed., pp. 528-529, 1940; MONTHLY WEATHER REVIEW 50:535-536, 1922, and 51:255-256, 1923. Cf. Besson, MONTHLY WEATHER REVIEW 51:254-255, 1923.

Planes	Refract- ing angles	Inclination of refract- ing edge to principal axis
Alternate faces of same pyramid (1, 3)	76°24'	42°48'
Opposite faces of same pyramid (1, 4)	49 42	90
A pyramidal face (1) and—		
Opposite face of hexagonal prism (4)	24 51	90
Alternate face of hexagonal prism (3)	63 01	28 08
A pyramidal face and base of hexagonal prism	65 09	90
A face of one pyramid (1') and alternate face of opposite pyramid (3)	53 58	28 08

An adequate explanation of a particular observed halo complex requires a physically probable crystal form to be found, of which a set of orientations likely to occur would have produced just the combination of arcs observed, and no others, and would consistently account for the relative intensities, details of coloring, and changes with time.

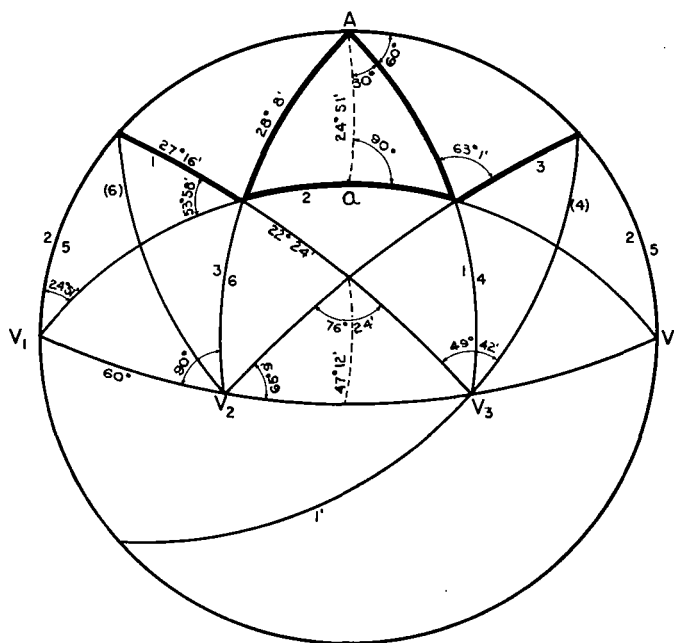


FIGURE 20.—Refracting angles in a pyramidal crystal. The figure is obtained by passing planes through the center of a sphere, parallel to the faces of a hexagonal right prism and the faces of a hexagonal right pyramid, cutting the sphere in great circles; the angles at which these circles intersect one another include all the dihedral angles formed by any two faces of any of the crystal forms in figure 19, and the points of intersection are the locations of the refracting edges. Pairs of opposite faces in the prism, or in a bipyramid, are called conjugate faces of the same great circle. In this example, $\alpha = 60^\circ$, $\beta = 70^\circ$, $V_a = 10^\circ$, $V_b = 60^\circ$ and $A_a = 24^\circ 51'$; any desired arc or angle in the figure may be computed.

The present series of papers is confined to only the geometrical formation of theoretically possible halos; but it is necessary to introduce a few physical considerations as a general guide. On any occasion, there will always be some crystals in each of all geometrically possible orientations in space. At times the distribution of the crystals will be entirely random—no larger proportion in one orientation than in any other; at other times, one or more particular orientations will more or less predominate. Each separate crystal will always produce a set of images, but whether or not the aggregate effect of all the crystals in the same orientation will be distinguishable depends jointly on the number and the proportion of crystals in this particular orientation and on the relative intensity of the light transmitted and reflected by that orientation (*cf.* paper I, p. 324).

In the formal geometrical theory, we shall take into account only those orientations of each crystal form that may reasonably be expected under natural conditions to lead to sufficient concentration of light for the production of readily observable effects: that is, only orientations that (1) correspond to the minimum minimorum, or (2) predominate as a result of a restrictive influence that deprives the crystal of one of its degrees of freedom and at the same time correspond to minimum deviation, or (3) predominate because of a restrictive influence that deprives the crystal of two of its degrees of freedom, in which case all deviations must be considered. In general, only reflections that are total need be taken into account.

The different orientations that are to be taken into account in any case, for deriving the collective effect of all the crystals, may be conveniently specified by the positions in which the principal crystallographic axis may lie in space, and the extent to which rotation of the crystal may take place around the axis.

THE OPTICAL METEORS PRODUCED BY CRYSTALS ORIENTED AT RANDOM

The case in which the crystals have three degrees of freedom and are oriented completely at random—as many crystals lying with their axes in one position as in any other, and rotating freely around their axes—is easily disposed of. The only important relative concentration of light into a limited region of the sky is produced by refraction at and very near the minimum minimorum.

Any plane through the line from observer to luminary will intersect some of the crystals; a certain proportion of these crystals will happen to be so oriented that the section by the plane is a principal plane of some one of the refracting angles, or very nearly so. All rays in this plane that are incident on such crystals will be refracted in or near a principal plane; the sections themselves will be randomly oriented in the intersecting plane, so that all possible values of the angle of incidence, and hence of the deviation, will occur. Of the crystals that produce any given deviation D , all those on a line through the observer at an angle D with the line from observer to luminary will send the refracted ray to the observer; the observer will therefore see an image on the sky at an angular distance

from the luminary equal to the deviation, and in a direction from the luminary on the great circle where the intersecting plane cuts the celestial sphere. The images corresponding to the different deviations in any such plane will collectively form an arc extending along this great circle from the minimum minimorum to the maximum deviation, but fading rapidly in brightness with increasing deviation. The same effect will be produced in all planes through the line from observer to luminary, all of which may be obtained by revolving a plane around this line; hence a circular ring of light will appear, centered at the luminary, with a sharp inner edge (contrasting with a comparatively dark sky within) of radius equal to the minimum minimorum, and a diffuse outer border merging into a general sky glare beyond.

The concentration of light near minimum deviation in the principal plane is so strong that these circular halos may be distinguishable even when particular orientations predominate among the crystals sufficiently to give other arcs also.

Each refracting angle can produce such a circular halo; and it is to phenomena of this type that the generic name halo properly applies (Gr., *ἅλως*). The radii of all these halos that can be produced by the crystal forms we have enumerated are as follows:

Refrac- tion angle	Radius of halo	Crystal elements required
° /	° /	
24 51	7 54	Hexagonal prism with pyramid.
49 42	17 06	Pyramid.
53 58	18 58	Bipyramid.
60 00	21 50	Hexagonal prism.
63 01	23 24	Hexagonal prism with pyramid.
65 09	24 34	Pyramid with plane base; or bipyramid with one truncation.
76 24	31 49	Pyramid.
90 00	45 44	Hexagonal prism, with plane base or truncated pyramid.

The 22° halo is by far the commonest of all halo phenomena; nearly all the others in this table have been observed with certainty, though most of them are very rare.³

³ See W. J. Humphreys, *Physics of the Air*, 3 ed., pp. 534-536, 1940, and the further references there given. Cf. Besson, MONTHLY WEATHER REVIEW, 42: 443, 1914, and 51: 254, 1923.

RECALIBRATION OF INSTRUMENTAL EQUIPMENT AT SOLAR RADIO STATIONS

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[U. S. Weather Bureau Solar Radiation Supervisory Station, Blue Hill Observatory of Harvard University, Milton, Mass., October 1941]

The desirability of recalibrating the equipment used at stations where records of solar radiation are now being made has long been recognized. The original calibrations of the pyrheliometers were made by three separate agencies: (1) the Solar Radiation Investigations section of the United States Weather Bureau (2) the Eppley Laboratory, and (3) the National Bureau of Standards. Calibrations by (1) were made by occulting the sun at regular intervals on clear days, subtracting the values of the sky radiation thus determined from the total radiation on a horizontal surface, and obtaining the ratio between this result and the otherwise measured value of the normal incidence radiation reduced to a horizontal surface by means of the sine law. Calibrations by (2) and (3) were obtained by direct comparison against standards furnished by the Weather Bureau. It was obvious that great improvement would be obtained if all instruments were recalibrated against a single carefully standardized pair of pyrheliometers; and the need for this increased after a

more thorough study of the Eppley pyrheliometer had shown that the cosine law failed to hold with low sun.¹ Moreover, some stations had not been inspected for over 10 years, and it was thought best to check not only the pyrheliometers but also the recording equipment and other accessories.

Between March and July 1941, all stations listed in table 1 were therefore visited; the pyrheliometers were carefully leveled, where necessary, and checked against either the 10- or the 50-junction standards, which previously had been standardized directly against the standard Smithsonian silver-disk normal incidence pyrheliometer. We may now be confident that all these stations are on the same standard, and as close to the Smithsonian scale of pyrheliometry as we are able to place it. Table 1 gives the average monthly c. m. f. of all pyrheliometers checked, and also the percentage change from the mean

¹ Byron H. Woertz and Irving F. Hand. The Characteristics of the Eppley Pyrheliometer. MONTHLY WEATHER REVIEW, 69: 146-148, 1941, May.